Exact differential equation

Ex. 1. $(x+y)dy - (x-y)dx = 0$

$\Rightarrow (x-y)dx - (x+y)dy = 0$

Here, $M = x-y$, $N = -(x+y)$

$\Rightarrow \frac{\partial M}{\partial y} = -1$, $\frac{\partial N}{\partial x} = -1$

Here, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow$ The given equation is exact.

Hence, the solution is

$\int M\,dx$ (Taking y-Constant) $+ \int N\,dy$ (terms not containing x)

$\Rightarrow \int (x-y)dx + \int -y\,dy$

$\Rightarrow \frac{x^2}{2} - xy - \frac{y^2}{2} = c$

$\Rightarrow x^2 - y^2 - 2xy = 2c$ Ans

2. (i) $(ax+hy+g)dx + (hx+by+f)dy = 0$

Here, $M = ax+hy+g$, $N = hx+by+f$

$\frac{\partial M}{\partial y} = h$, $\frac{\partial N}{\partial x} = h$

$\Rightarrow$ The given eq? is exact.

Hence, the solution is

$\int (ax+hy+g)dx$ (taking y-Constant) $+ \int (by+f)dy$ (terms not containing x)

3. $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$

Here, $M = x^2 - 4xy - 2y^2$, $N = y^2 - 4xy - 2x^2$

$\frac{\partial M}{\partial y} = -4x - 4y$ $\quad \frac{\partial N}{\partial x} = -4y - 4x$

$\Rightarrow$ The given equation is exact.

Hence, the solution is

$\int (x^2 - 4xy - 2y^2)dx + \int y^2 dy$

$\Rightarrow \frac{x^3}{3} - \frac{2 \cdot 4y \cdot x^2}{2} - 2xy^2 + \frac{y^3}{3} = k$

$\Rightarrow x^3 + y^3 - 6xy(x+y) = c$ Ans.

4. $(2x - y + 1)dx + (2y - x - 1)dy = 0$

Here, $M = 2x - y + 1$, $N = 2y - x + 1$

$\frac{\partial M}{\partial y} = -1$ $\quad , \frac{\partial N}{\partial x} = -1$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow$ The given equation is exact.

Hence, the solution is

$\int (2x - y + 1)dx + \int (2y - 1)dy$

$\Rightarrow x^2 - xy + x + y^2 - y = c$ Ans.

~~$\Rightarrow x^2 =$~~

5. $\frac{dH}{dx} = \frac{6x - 2y - 7}{2x + 3y - 6}$

$\Rightarrow (6x-2y-7)dx - (2x+3y-6)dy = 0$

Here, $M = 6x-2y-7$, $N = -(2x+3y-6)$

$\Rightarrow \frac{\partial M}{\partial y} = -2$ $\quad \frac{\partial N}{\partial x} = -2$

$\Rightarrow$ The given equation is exact.

Hence, the solution is

$\int (6x-2y-7)dx + \int (-3y+6)dy = 0$

y-Constant

$\Rightarrow 6\frac{x^2}{2} - 2y.x - 7x - \frac{3y^2}{2} + 6y = C.$

$\Rightarrow 3x^2 - 2xy - 7x - \frac{3}{2}y^2 + 6y = C$ Ans

6. $(2x+3y-5)dy + (3x+2y-5)dx = 0$

Here, $M = 3x+2y-5$, $N = 2x+3y-5$

$\frac{\partial M}{\partial y} = 2$ $\quad \frac{\partial N}{\partial x} = 2$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\Rightarrow$ The given equation is exact

Hence, the solution is

$\int (3y-5)dy + \int (2y-5+3x)dx$

y-constant

$\Rightarrow \frac{3}{2}y^2 - 5y + 2xy - 5x + \frac{3}{2}x^2 = K$

$\Rightarrow 3(x^2+y^2) + 4xy - 10(x+y) = 2K = C$ Ans